Digital representation workshop

Bits and bytes and all that

Prerequisites

We assume that you have read the following prior to attending this workshop:

- Kernigan - Chapter 2 - “Bits, Bytes and Representation of Information” (pp. 21-34)
- Juneday wiki - http://wiki.juneday.se/mediawiki/index.php/Computing:Binary_representation

And seen:

- Binary Representation (Full playlist) Binary Representation - Text (Full playlist)

Objectives

Understand

- how information is represented in computers
- the binary number system
- how numbers can be used to represent anything - like text for instance

Know

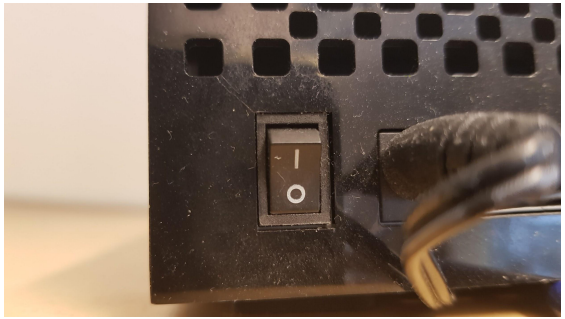
- how to represent text as binary numbers
- how to convert between different bases

Divide yourselves in groups

- 4-5 persons in each group
- You will work together during the workshop

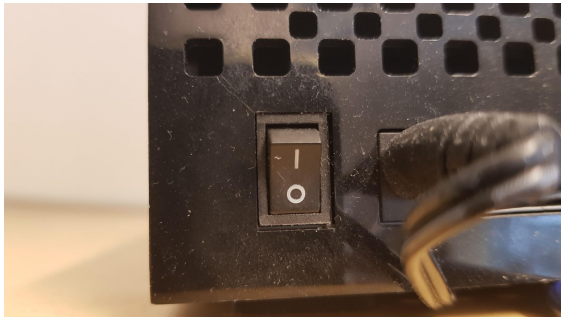
Binary basics

- How much information can we store in one bit (0|1)
  - Why?
  - Examples of one bit information
- Number of combinations in *n* bits
  - How many combinations are there in 10 bits?
- How many bits do we need to store one million numbers?
- A ball park figure for 2^{32} ?
  - Recap of multiplication of exponentiations, e.g. $2^n 2^m = 2^{n+m}$
- Discuss in your groups and agree on an answer



Binary basics - suggested solution

- How much information can we store in one bit (0|1)
  - Why?
  - Examples of one bit information
- We can store two states, like On/Off, odd/even etc
- This is because 2^1 is two



Binary basics - suggested solution

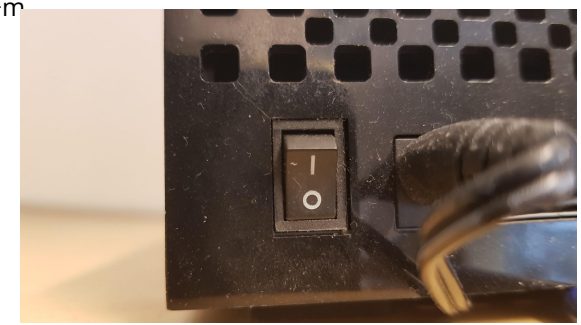
- Number of combinations in *n* bits
  - How many combinations are there in 10 bits?
- 10 bits can represent 1024 different things, like numbers. Remember this number, it is good for estimates.
- $2^4 = 16$.
  $2^6 = 64$.
  $2^{10} = 2^4 * 2^6 = 2^{4+6}$.
  $2^4 * 2^6 = 16 * 64 = 1024$.
- Eller:
- 2*2*2*2*2*2*2*2*2*2 = 1024.

Binary basics - suggested solution

- How many bits do we need to store one million numbers?
- We need 20 bits, since 2^{20} is the smallest power of two, that is larger than or equal to one million. How could we use previous knowledge to solve this?
- We learnt that 2^{10} was 1024. We know that 1024 * 1024 ≈ 1 000 000.
- We know that $2^{20} = 2^{10} * 2^{10}$.
- Answer: 20 bits can represent over 1 000 000 numbers.
- Exact answer: 20 bits can represent 1 048 576 numbers.

Binary basics - suggested solution

- A ball park figure for 2^{32} ?
  - Recap of multiplication of exponentiations, e.g. $2^n 2^m = 2^{n+m}$
- About 4 000 000 000, because:
- $2^{10} * 2^{10} * 2^{10} * 2^2 = 2^{10+10+10+2}$.
- $2^{10} * 2^{10} * 2^{10} \approx 1\ 000\ 000\ 000$.
- $2^2 = 4$.
- Answer: about 4 000 000 000
- Exact answer: 4 294 967 296



Binary numbers - converting to decimal

- Just like decimal numbers, the most significant digit is to the left.
- Using four bits for unsigned numbers, the rightmost bit is worth 2^0 (1_{10}) if set. The next 2^1 and the next 2^2 and the next (leftmost) 2^3.

```
1010
|||` 0 * 2^0
||`- 1 * 2^1
|`-- 0 * 2^2
`--- 1 * 2^3
```

Sum: $0_{10} + 2_{10} + 0_{10} + 8_{10} = 10_{10}$

Binary numbers - converting to decimal

- Figure out what the following unsigned four bit numbers represent converted to decimal:
- 1111
- 1110
- 1101
- 1100
- 1011
- 1010
- 1001
- 1000

Binary numbers - converting to decimal - answers

- $1111_2 = (1 + 2 + 4 + 8 = 15)_{10}$
- $1110_2 = (0 + 2 + 4 + 8 = 14)_{10}$
- $1101_2 = (1 + 0 + 4 + 8 = 13)_{10}$
- $1100_2 = (0 + 0 + 4 + 8 = 12)_{10}$
- $1011_2 = (1 + 2 + 0 + 8 = 11)_{10}$
- $1010_2 = (0 + 2 + 0 + 8 = 10)_{10}$
- $1001_2 = (1 + 0 + 0 + 8 = 9)_{10}$
- $1000_2 = (0 + 0 + 0 + 8 = 8)_{10}$
- How many bits do we have to investigate, in order to tell if any binary number is odd or even?

Binary numbers, convert from decimal

Calculate the four bit unsigned representation (pad with 0s if necessary) of the following unsigned numbers:

- 5
- 3
- 7
- 6
- 0
- 2

Binary numbers, convert from decimal - answers

- $5_{10} = (0101)_2$
- $3_{10} = (0011)_2$
- $7_{10} = (0111)_2$
- $6_{10} = (0110)_2$
- $0_{10} = (0000)_2$
- $2_{10} = (0010)_2$

7/2 = 3, remainder 1, so least significant bit is 1 -> 1

3/2 = 1, remainder 1, so next bit is 1 -> 11

1/2 = 0, remainder 1, so next bit is 1 -> 111

Pad with 0 -> 0111

Bonus - bash one liner for eight bit binary

```
i=111; result=$(while ((i/2 > 0));do echo -n $((i % 2));i=$i/2;done;echo $((i %2)));printf
%08d $(echo $result|rev);echo

# formatted:

i=111;
result=$(
  while ((i/2 > 0));
  do
    echo -n $((i % 2));
    i=$i/2;
  done;
echo $((i %2)));
printf %08d $(echo $result|rev);
echo
```

Over- and underflow

If we are using 8 bits to represent numbers, what happens when we add such numbers together?

Largest unsigned number using 8 bits is $1111\ 1111_2$ (255_{10}).

What happens when we add $0000\ 0011_2$ (3_{10}) to it?

Remember how to add binary numbers:

0+0 = 0
0+1 = 1
1+0 = 1
1+1 = 10 (one in carry)

Over- and underflow

If we are using 8 bits to represent numbers, what happens when we add such numbers together?

Largest unsigned number using 8 bits is $1111\ 1111_2$ (255_{10}).

What happens when we add $0000\ 0011_2$ (3_{10}) to it?

```
11111 111
 1111 1111
+0000 0011
----------
 0000 0010 ← uh-oh
```

Signed numbers (2's complement)

- In signed numbers (using 2's complement) the leftmost bit carries the sign
  - 1 means negativ number, 0 means positive
  - We'll "loose" one bit, the range becomes -2^{n-1} .. $2^{n-1}-1$ (n is the number of bits)
- Overflow may change the sign
- 2's complement: if negative, flip the bits and add one
- Example: four bits, -8_{10} is:

```
1000 ("normal" 8)
0111 (flipped)

 0111
+0001
 1000 (one added)
```

Signed numbers overflow

Using 2’s complement, what is 7+2 in binary?

```
 0111
+0010
 ????
```

Signed numbers overflow

Using 2’s complement, what is 7+2 in binary?

```
 0111
+0010
 1001
Negative, so flip bits and add one:
 0110
 0001
 0111 (7, so it represents -7)
```

Representing text in (8 bit) binary format

Characters in a computer can be assigned a number (e.g. ASCII table).

Therefore, we can represent text as a sequence of such numbers binary.

Excerpt (using 8 bit ascii):

```
Binary    Oct  Dec  Hex       Glyph
0000 1010 012  10   0A        <Line Feed>
...
0110 0001 141  97   61        a
0110 0010 142  98   62        b
0110 0011 143  99   63        c
0110 0100 144  100  64        d
0110 0101 145  101  65        e
```

Representing text in (8 bit) binary format

Group exercise: What text does the following binary information represent?

```
01000010 01101001 01101110 01100001 01110010 01111001
00100000 01101110 01110101 01101101 01100010 01100101
01110010 01110011 00100000 01100001 01110010 01100101
00100000 01100110 01110101 01101110 00101110 00001010
01000001 01110010 01100101 01101110 00100111 01110100
00100000 01110100 01101000 01100101 01111001 00111111
```

Hint: https://en.wikipedia.org/wiki/ASCII (leading zeros don't show in the table)

Hint: install the ascii and ascii2binary commands

Representing text in (8 bit) binary format

Group exercise: What text does the following binary information represent?

```
01000010 01101001 01101110 01100001 01110010 01111001
00100000 01101110 01110101 01101101 01100010 01100101
01110010 01110011 00100000 01100001 01110010 01100101
00100000 01100110 01110101 01101110 00101110 00001010
01000001 01110010 01100101 01101110 00100111 01110100
00100000 01110100 01101000 01100101 01111001 00111111

$ for b in 01000010 01101001 01101110 01100001 01110010 01111001 00100000 01101110 01110101
01101101 01100010 01100101 01110010 01110011 00100000 01100001 01110010 01100101 00100000
01100110 01110101 01101110 00101110 00001010 01000001 01110010 01100101 01101110 00100111
01110100 00100000 01110100 01101000 01100101 01111001 00111111;do echo "$b"|ascii2binary -b
b;done;echo
Binary numbers are fun.
Aren't they?
```